Clint

GRUMMAN AEROSPACE CORPORATION

LM MEMORANDUM

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LMO-500-767

From: R. Schindwolf RJ

To: Distribution

Subject: Rate of Descent (ROD) Mode Stability and Performance Analysis.

SUMMARY:

The results of this analysis indicates that the stability of the ROD control system can be optimized by properly matching the LGC programmed gain constants to the system time delays. For Apollo 12 the engine response time assumed in choosing the gain constants was longer than the actual engine response time. This resulted in a lightly damped system and in conjunction with the extraneous accelerations introduced by the IMU mounted off the C.G. resulted in the large throttle oscillations observed during the Apollo 12 landing. The throttle oscillations observed during FCI lab. tests were due to accelerometer noise introduced by quantization in the PIPA math model. To reduce the sensitivity of the loop to extraneous accelerations it is recommended that the LGC gain constants $\frac{LAG}{\tau}$ and τ_{t} be changed to 0.2 and .1 seconds respectively. These gains will result in a proper match to the best estimate of the system time delays. In addition it is recommended that the program change suggested by MIT, which approximately compensates for the extraneous accelerations introduced by the off-C.G. IMU be incorporated. These changes will reduce the peak to peak throttle variations from 2500 pounds as observed on Apollo 12 to approximately 300 - 400 pounds.

INTRODUCTION:

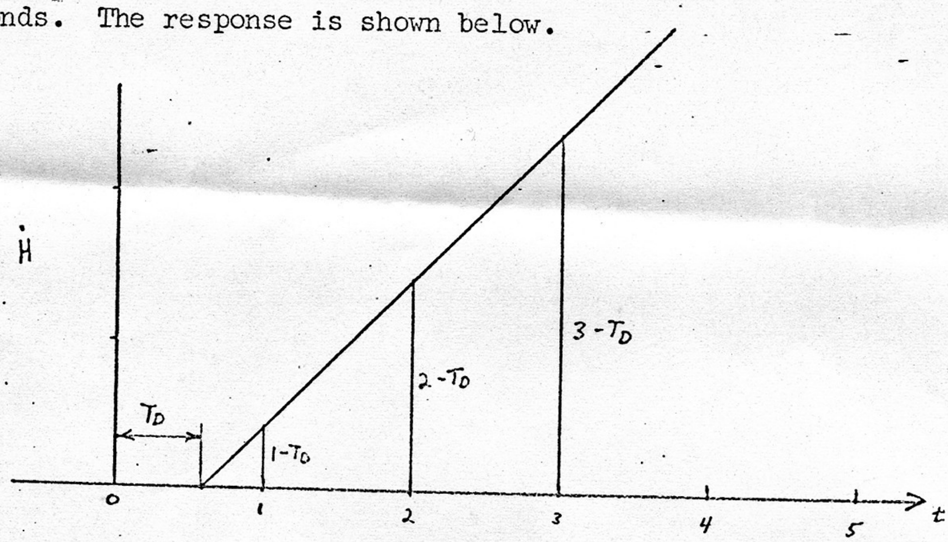
FMES/FCI powered descent tests indicated that throttle oscillations in the order of 800 pounds peak to peak occurred during the ROD mode. A review of the Apollo 12 flight data indicated that throttle oscillations as large as 2500 pounds peak to peak occurred during ROD operation. The purpose of this analysis was to determine if these oscillations were due to an inherent instability in the ROD control loop or to extraneous forcing functions.

DISCUSSION:

A loop diagram of the ROD throttle loop is shown in Figure 1. This loop was derived from the Guidance Equation Section of the GSOP. The e^{-ST} term used in the transfer functions represents a time delay of one computation

period (1 second). Due to the sample data nature of the system it is more convenient to use Z transforms instead of Laplace transforms ($Z = e^{ST}$). The loop diagram using Z transforms is shown in Figure 2.

The Z transform from ΔA_c^* to $\dot{H}$ is not readily apparent but can be deduced by physical reasoning. The Z transform of a linear system is simply the response of the system at the sampling times to an input impulse. A unit impulse in ΔA_c^* will result in an $\dot{H}$ ramp of unit slope and delayed by T_D seconds. The response is shown below.



The Z transform is therefore

$$\frac{\dot{H}(z)}{\Delta A(z)} = \frac{1-T_D}{Z} + \frac{2-T_D}{Z^2} + \frac{3-T_D}{Z^3} + \cdots$$

$$= \left[\frac{1}{Z} + \frac{2}{Z^2} + \frac{3}{Z^3} + \cdots\cdots\right] - T_D\left[\frac{1}{Z} + \frac{1}{Z^2} + \frac{1}{Z^3} + \cdots\cdots\right]$$

$$= \frac{Z}{(Z-1)^2} - T_D\frac{1}{Z-1} = \frac{(1-T_D)Z + T_D}{(Z-1)^2}$$

System stability is determined by the location of the roots of $1 + G(Z)$ where $G(Z)$ is the open loop gain function $\frac{\dot{H}(Z)}{\Delta\dot{H}(Z)}$. For the system to be stable the roots of $1 + G(Z)$ must lie inside the unit circle in the Z plane. $G(Z)$ can be obtained from the loop diagram and is as follows:

$$G(z) = \frac{1}{J}\,\frac{Z\left[(1-T_D)Z + T_D\right]}{(Z-1)^2\left[Z + \left(1+\frac{LAG}{J}\right)T_D'\right] + \left(1+\frac{LAG}{J}\right)(Z-1)\left[(1-T_D)Z + T_D\right]}$$

where T_D = total system time delay including computational time and engine delays.

T_D' = Computer estimate of total time delay

$= (t_{fc} - t_n) + \tau_{th} + \frac{(f - \bar{f})/FRATE}{2}$

Therefore

$$1 + G(z) = \frac{(z-1)^2\left[z + \left(1 + \frac{LAG}{\tau}\right)T_D'\right] + \left(1 + \frac{LAG}{\tau}\right)(z-1)\left[(1-T_D)z + T_D\right] + \frac{1}{\tau}z\left[(1-T_D)z + T_D\right]}{(z-1)^2\left[z + \left(1 + \frac{LAG}{\tau}\right)T_D'\right] + \left(1 + \frac{LAG}{\tau}\right)(z-1)\left[(1-T_D)z + T_D\right]}$$

and system stability is determined by the zeros of the polynomial

$$(z-1)^2\left[z + \left(1 + \frac{LAG}{\tau}\right)T_D'\right] + \left(1 + \frac{LAG}{\tau}\right)(z-1)\left[(1-T_D)z + T_D\right] + \frac{1}{\tau}z\left[(1-T_D)z + T_D\right]$$

Rearranging terms this polynomial can be written as follows:

$$z^2\left(z - \frac{\tau-1}{\tau}\right) + \frac{LAG - T_D}{\tau}(z-1)z + (T_D' - T_D)\left(1 + \frac{LAG}{\tau}\right)(z-1)^2$$

If LAG and T_D' are properly matched to the system delays ($LAG = T_D' = T_D$) the last two terms drop out and the characteristic equation is

$$z^2\left(z - \frac{\tau-1}{\tau}\right)$$

With $\tau = 1.5$ this becomes

$$z^2(z - .33)$$

Obviously this system is extremely stable with a double root at the origin and a root at + .33.

In order to determine the effects of mismatches the root loci of Figures 3, 4 and 5 were drawn. Figure 3 gives the root locus as LAG is varied from -1.2 to +1.2. In this diagram it is assumed that the system time delay, T_D, is .3 seconds and that T_D' is properly matched. From this locus it can be seen that any value of LAG other than .3 (the matched value) will result in the root moving away from the origin toward the unit circle and will be less stable. Figures 4 and 5 give the same loci except with T_D' mismatched by + .1 seconds (over compensation) and -.1 seconds (under compensation) respectively.

It can be seen that the T_D' mismatch (both positive and negative) also causes a reduction in stability since the roots are further from the origin than in Figure 3. It is therefore concluded that by choosing LAG and T_D to be equal to the best estimate of the system time delay, optimum stability will be achieved. Since the best estimate of total system time delay is approximately .3 seconds (.2 seconds computation time and .1 seconds throttle lag) the $\frac{LAG}{\tau}$ term in the LGC program should be 0.2. The LGC determines T_D' by measuring the computational delay and adding a prestored value of throttle lag (τ_{th}). For optimum stability this stored value should equal the best estimate of throttle lag (.1 seconds).

The stored values of $\frac{LAG}{\tau}$ and τ_{th} used on Apollo 12 were 0.4 and 0.2 seconds respectively. These values result in the root locations circled in Figure 4. It can be seen that one of the roots is close to the unit circle resulting in a lightly damped system. It is therefore recommended that for future flights the value of $\frac{LAG}{\tau}$ and τ_{th} be changed to 0.2 and 0.1 seconds respectively.

In order to explain the large throttle variations the response of the system to accelerometer noise was computed. The closed loop transfer function relating acceleration command to accelerometer ΔV errors is as follows:

$$\frac{A_c}{\varepsilon_V} = \frac{Z(Z-1)\left[\left(1+\frac{LAG}{\tau}+\frac{1}{\tau}\right)Z-\left(1+\frac{LAG}{\tau}\right)\right]}{Z^2\left(Z-\frac{\tau-1}{\tau}\right)+\frac{LAG-T_D}{\tau}(Z-1)Z+(T_D'-T_D)\left(1+\frac{LAG}{\tau}\right)(Z-1)^2}$$

The transfer functions using the properly matched gains and the Apollo 12 gains are as follows:

$$\frac{A_c}{\varepsilon_V} = \frac{1.87Z^3-3.07Z^2+1.2Z}{Z^3-.33Z^2} \qquad \text{(Matched)}$$

$$\frac{A_c}{\varepsilon_V} = \frac{2.07Z^3-3.47Z^2+1.4Z}{Z^3+.01Z^2-.48Z+.14} \qquad \text{(Apollo 12)}$$

The acceleration command resulting from a one foot per second velocity error impulse is obtained by expanding these transfer functions as power series in $\frac{1}{Z}$. The two responses are plotted in Figure 6.

The Apollo 12 response is particularly significant since the magnitude of the velocity errors introduced by the off-CG IMU was in the order of 1 ft/sec. From the plot it can be seen that the peak to peak acceleration command variation is as large as 5.9 ft/sec^2 which is equivalent to 3000 lbs of thrust variation. This agrees quite well with the thrust variations observed on Apollo 12. In addition, the response indicates poor damping as expected from the stability analysis. The response for the properly matched system shows similar peak amplitudes but decays rapidly after the first two sample periods.

These response characteristics can also be used to explain the throttle variations observed in FMES/FCI lab tests. In the FMES there are no velocity errors due to an off-C.G. IMU but errors are introduced by 50 millisecond time quantization of the simulated accelerometer outputs. This can result in instantaneous velocity errors as large as .25 ft/sec and peak to peak throttle variations in the order of 700 pounds for the Apollo 12 gains. This agrees quite well with the results observed in tests.

The throttle variations in the ROD mode can be reduced by either compensating for the erroneous accelerometer data or by changing the control loop to make it less sensitive to short term acceleration errors. A program modification that compensates for the errors due to the off-C.G. IMU has been developed by MIT and tested in the FCI lab. The test results indicated throttle variations of 400 lbs under conditions similar to those experienced in Apollo 12. In order to reduce the system sensitivity it is desirable to make the gain changes mentioned previously to match the system time delays. This does not change the initial amplitude appreciably but it does reduce the settling time. Other values of LAG and throttle lag time may result in somewhat smaller initial response but will result in reduced damping and are therefore not desirable. One method of reducing the initial amplitude and still maintaining good damping is to increase τ. The response to a unit velocity impulse for $\tau = 3.0$ is shown in Figure 7. It can be seen that the peak to peak response is reduced by about 30%. The disadvantage of this gain change, however, is that it results in increased response time to ROD inputs.

In order to obtain a significant reduction in accelerometer noise sensitivity without increasing ROD response time it is necessary to modify the basic loop configuration. The most significant feedback paths for erroneous acceleration data is through the term $\left(|AF| + \frac{\delta f_p}{m}\right)$ which is used in the P66 guidance routine and in the throttle command routine. This term is essentially the present value of vehicle thrust acceleration. Rather than using accelerometer derived data this term can be replaced by the commanded acceleration from the previous cycle. All other gains and parameters would be unchanged. This will eliminate two accelerometer feedback paths and therefore reduce considerably the response to accelerometer noise. The response to ROD inputs and system stability will not be affected by this change. The response of this configuration to a one ft/sec accelerometer error is shown in Figure 8 and it can be seen that the peak to peak response has been reduced by a factor of about 4. This is a significant reduction and it is recommended that the program changes necessary to implement this technique be evaluated.

CONCLUSIONS:

The throttle oscillations observed on Apollo 12 were due to the control system being lightly damped and being forced by exteraneous accelerations due to the off-C.G. IMU. To improve the damping and stability of the system it is recommended that the LGC program constants $\frac{LAG}{\tau}$ and τ_{th} be changed to .2 and .1 respectively. In addition, the MIT program change which approximately compensates for the offset IMU accelerations should be incorporated. The alternate loop configuration suggested should be studied further since it will reduce the loop response to any other extraneous acceleration data such as PIPA quantization, time quantization, vibration, etc.

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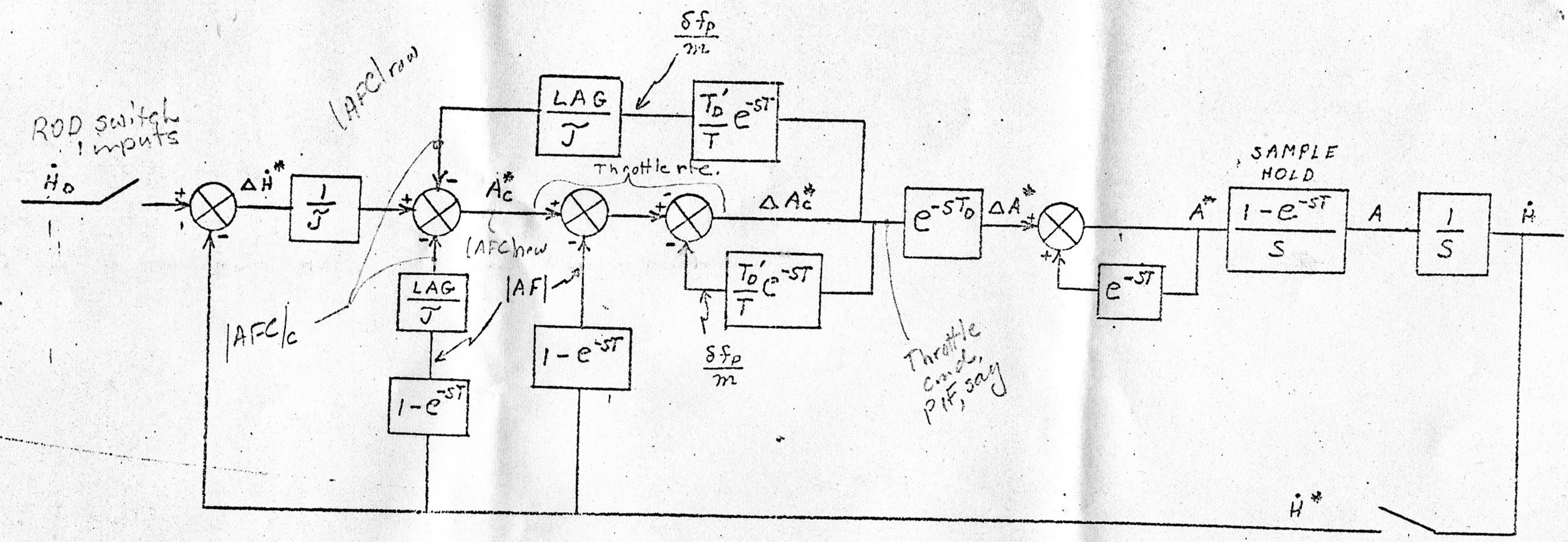


T_D = System time delay including computation delay and throttle response lags

T_D' = Computer estimate of system time delays = $(t_{fc} - t_n) + \mathcal{J}_{th} + \dfrac{(f - \bar{f})/\text{FRATE}}{2}$

T = Computer cycle time (1 second)

LAG, $\mathcal{J}$ = System gain constants

*starred parameters represent sampled quantities

FIGURE 1 - RATE OF DESCENT LOOP

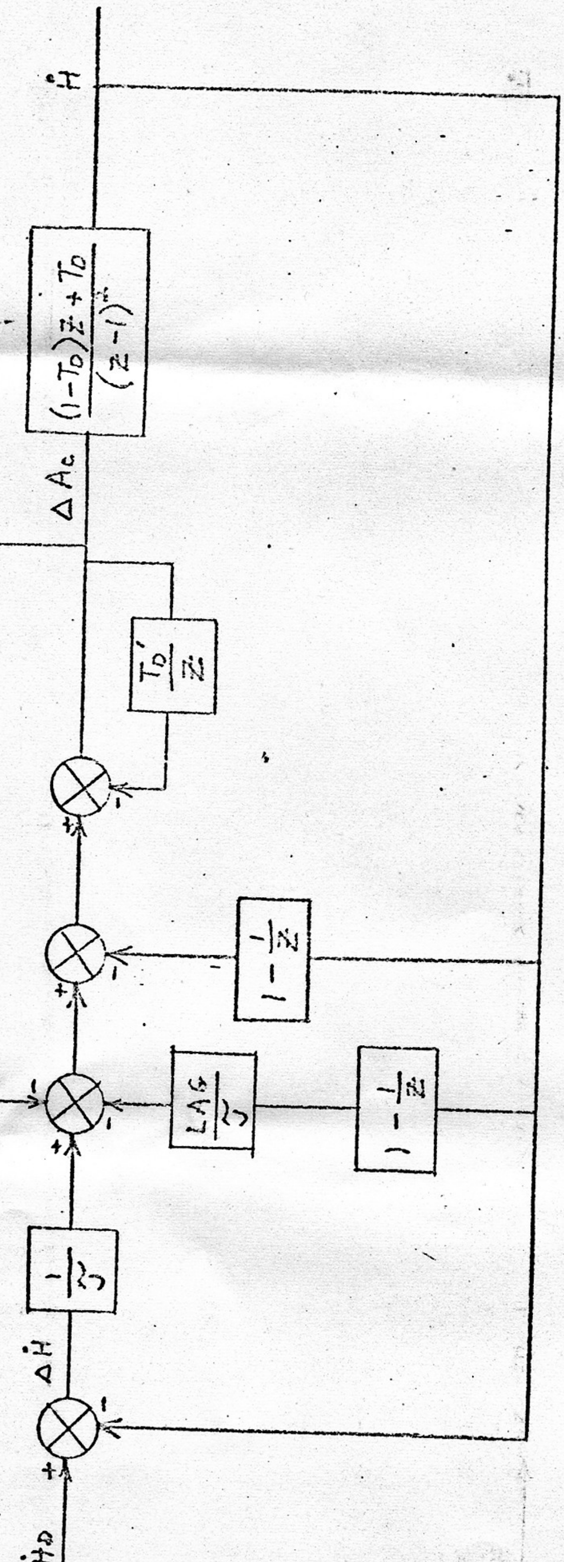


FIGURE 2 - Z TRANSFORMED ROD LOOP

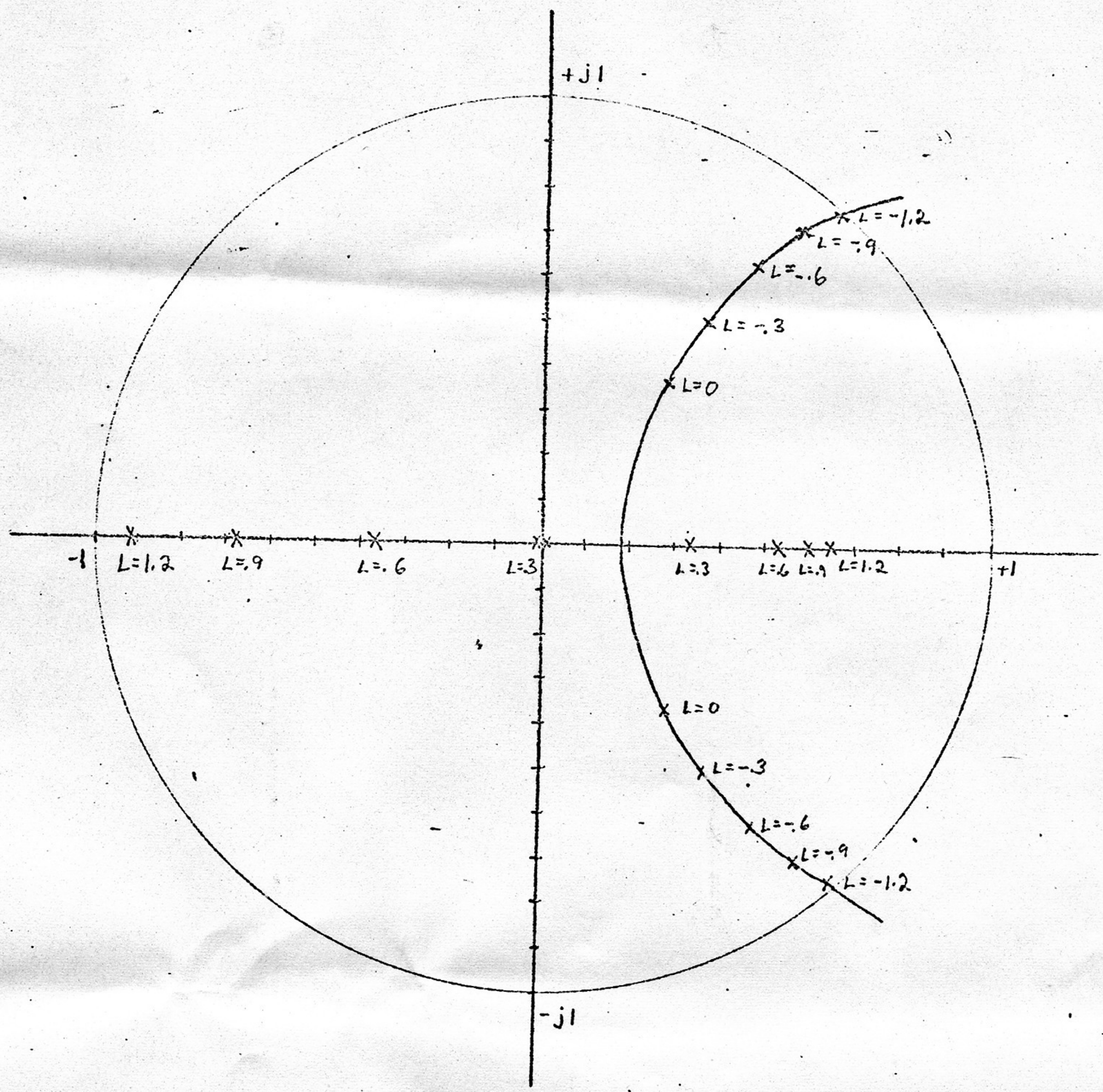


FIGURE 3 - Z PLANE ROOT LOCUS ($T'_D - T_D = 0$)

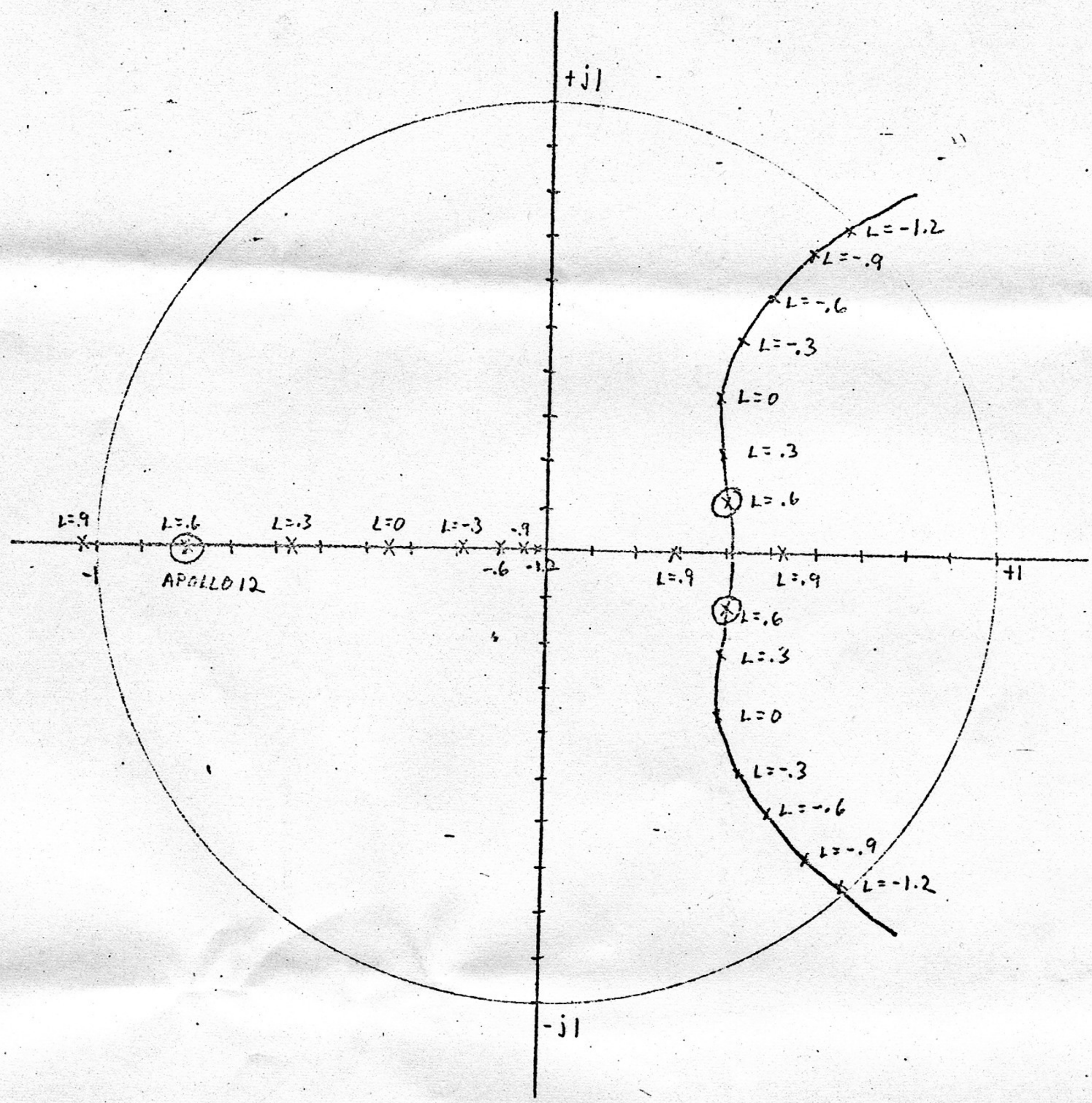


FIGURE 4 - Z PLANE ROOT LOCUS ($T_D' - T_D = +.1$)

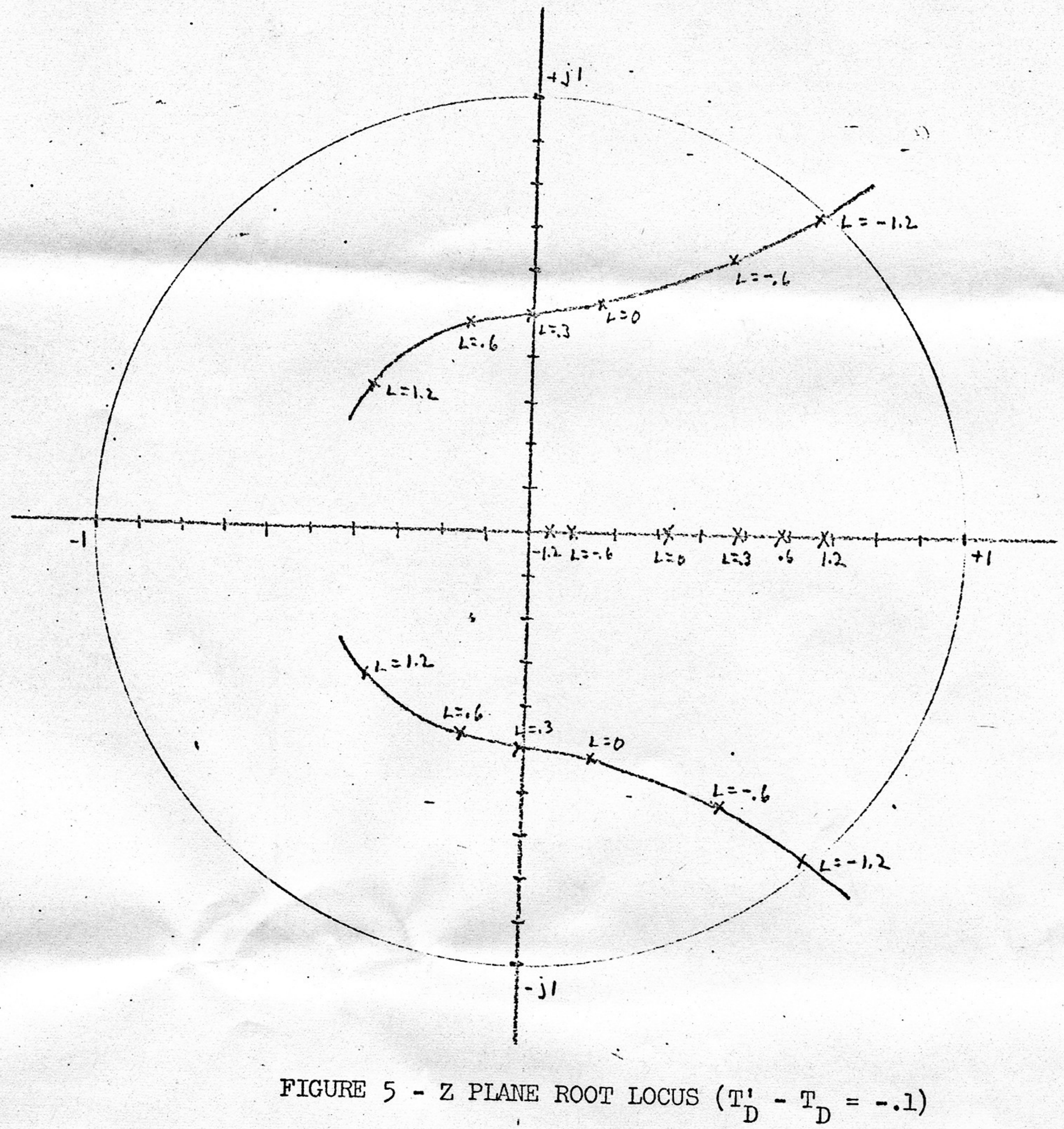
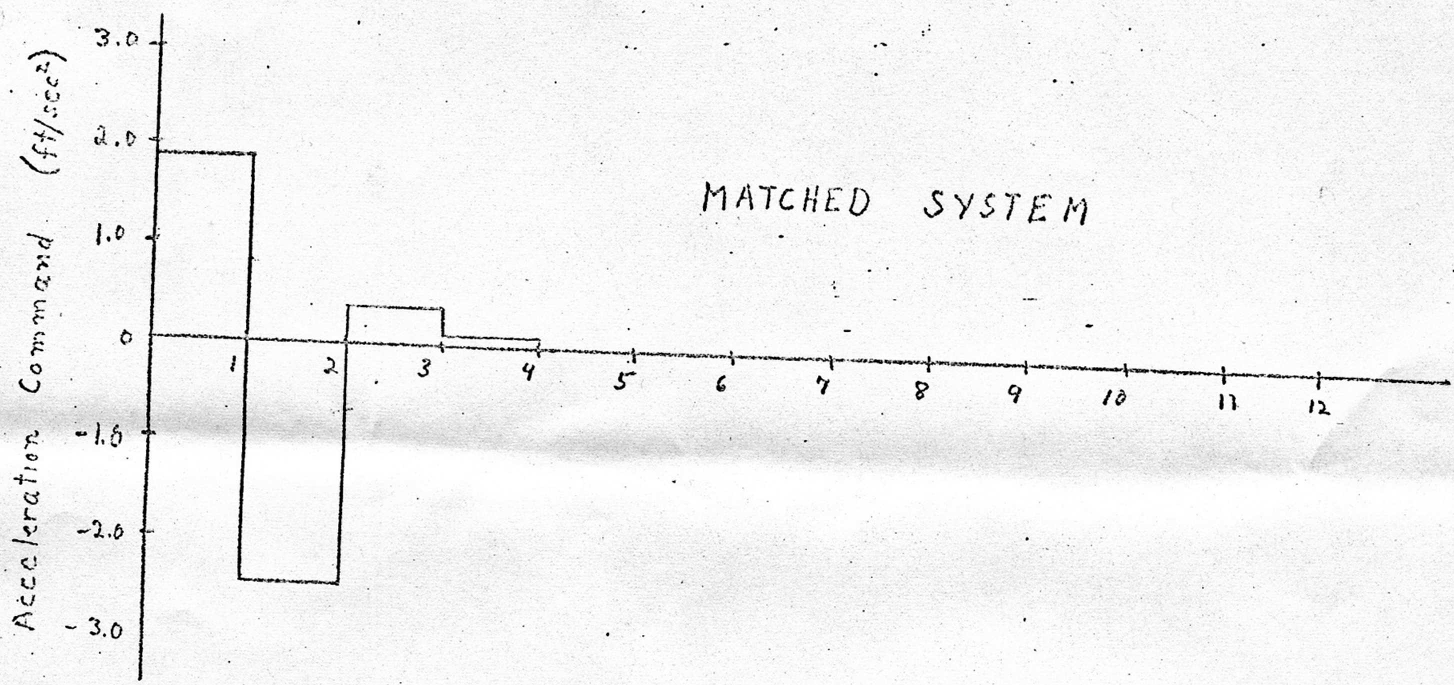


FIGURE 5 - Z PLANE ROOT LOCUS ($T_D' - T_D = -.1$)



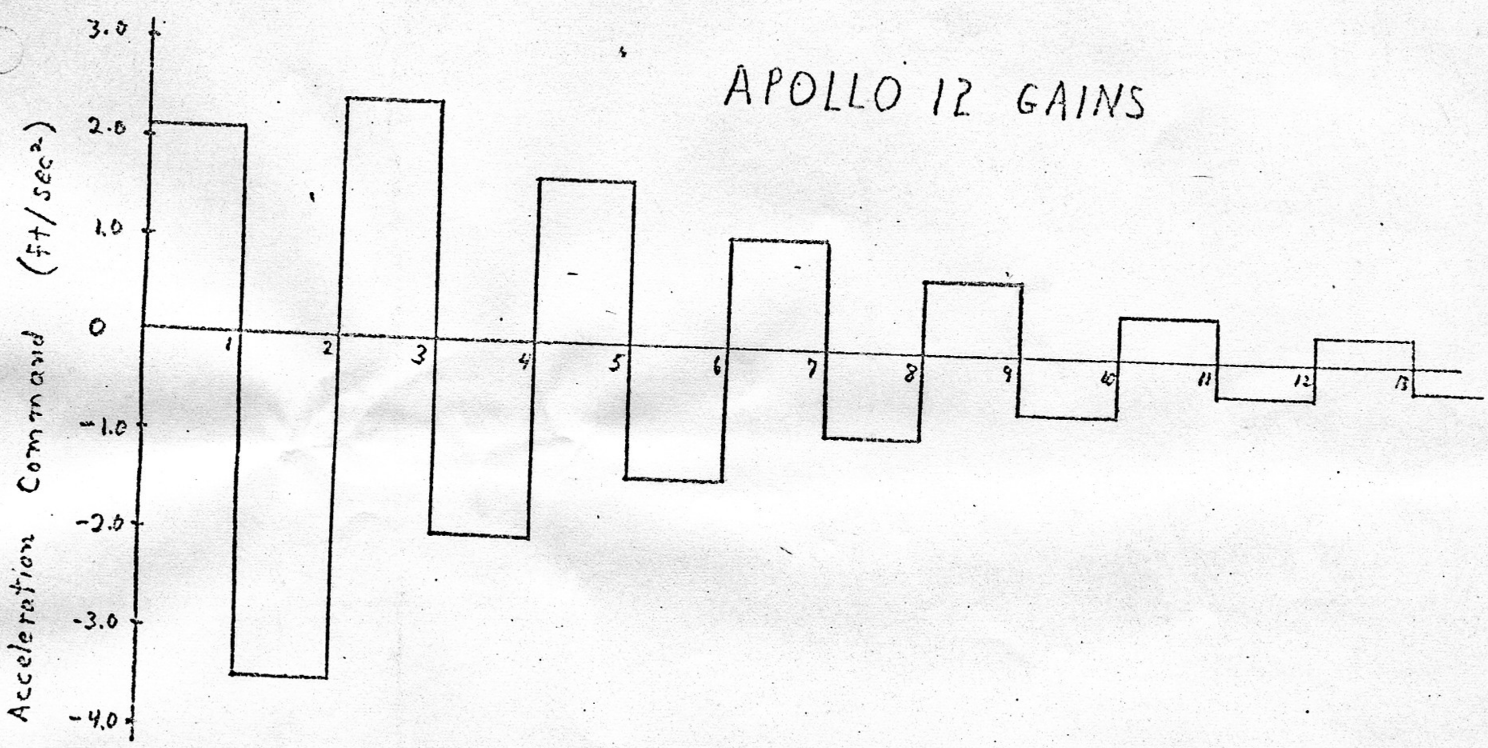


FIGURE 6 - RESPONSE TO 1 FT/SEC. ACCELEROMETER IMPULSE.

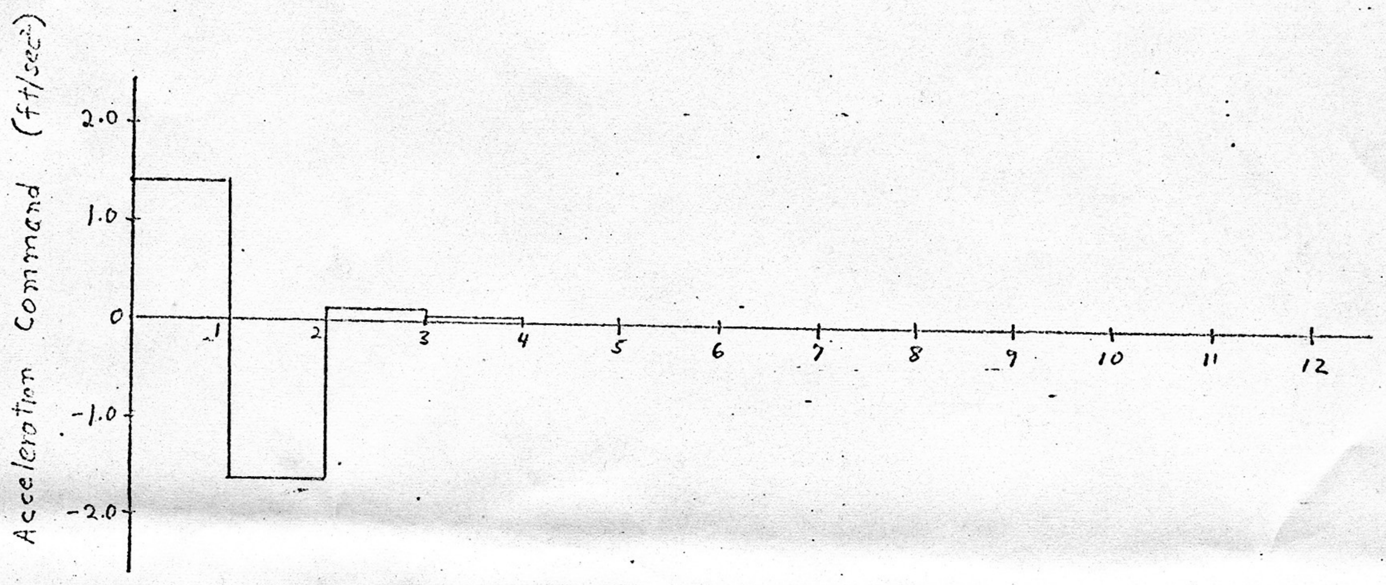


FIGURE 7 - RESPONSE TO 1 FT/SEC ACCELEROMETER IMPULSE

$\tau = 3.0$

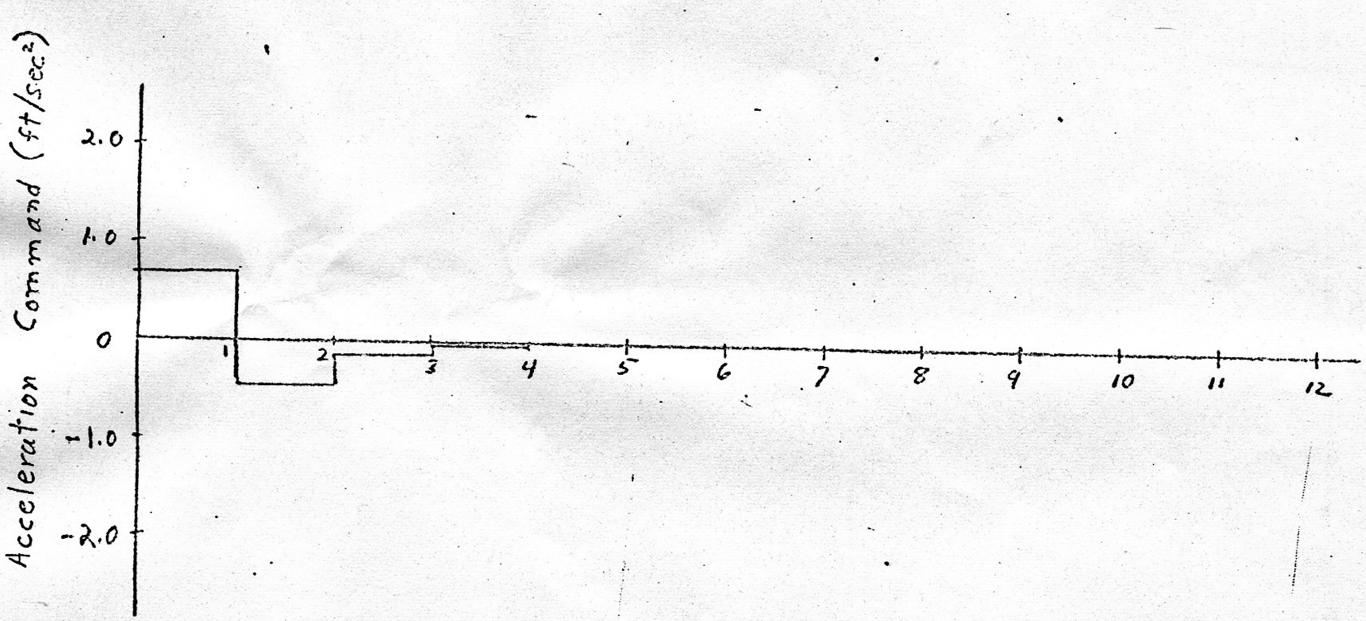


FIGURE 8 - RESPONSE TO 1 FT/SEC ACCELEROMETER IMPULSE

$\left(|AF| + \frac{\delta fp}{m}\right)$ REPLACED BY $A_{c_{n-1}}$